A Phenomenology of Galileo's Experiments with Pendulums

Supporting Document

ABBREVIATIONS

EN The so-called *Edizione Nazionale* of Galileo's works in twenty volumes (G. Galilei, *Le opere di Galileo Galilei. Edizione Nazionale* [ed. A. Favaro], 20 vols., Florence, 1890-1909). I quote this edition as EN, followed by a Roman numeral indicating the volume, and by Arabic numerals indicating page numbers.

RGE P. Palmieri, *Reenacting Galileo's Experiments: Rediscovering the Techniques of Seventeenth-Century Science*, Lewiston, NY, 2008.

T1 (EN, X, pp. 97-100). Letter of Galileo to Guido Ubaldo dal Monte. Padua, 29 November 1602. (Trans. in RGE, pp. 257-260).

T2 (EN, VII, pp. 256-257). Excerpt from the *Dialogue on the two chief world systems (1632)*. (Trans. in RGE, pp. 260-262).

T3 (EN, VII, pp. 474-476). Excerpt from the *Dialogue on the two chief world systems (1632)*. (Trans. in RGE, pp. 262-263).

T4 (EN, VIII, pp. 128-129). Excerpt from the *Two new sciences (1638)*. (Trans. in RGE, pp. 263-264).

T5 (EN, VIII, pp. 139-140). Excerpt from the *Two new sciences (1638)*. (Trans. in RGE, pp. 264-265).

T6 (EN, VIII, pp. 277-278). Excerpt from *Two new sciences (1638)*. (Trans. in RGE, pp. 265-268).

1. Artefacts in Naylor’s pendulum setup

There is no question that it is a little bit of a puzzle that a brass pendulum should be faster for smaller arcs, while the opposite seems to be true for the cork pendulum. So, how can the cork pendulum apparently violate the regularity of simple oscillatory motions? We know that isochronism is only an approximation to the regularity of simple oscillatory motions, but it is hard to see why a cork pendulum should be faster for larger arcs. Naylor does not attempt to explain this fact. He was clearly baffled by the situation.

I think that this baffling situation is due to possible artefacts in Naylor’s setup. In order to substantiate my conclusion, I will first discuss new findings based on an analysis of pendular motion carried out by exploring a wide range of parameters with a computer model. I will subsequently speculate about the reasons for the artefact in Naylor’s setup.

All we know about Naylor’s setup is that his cork pendulum was 81 inches long. Naylor does not report the mass of the cork bob, the weight and size and/or material of the string by which the pendulum was suspended (if indeed he used a string). Naylor does not report the amplitude of the arcs of oscillation of the pendulums. All he says is that it proved easy to measure times of oscillations of the brass pendulum for arcs between 160 and 120 degrees. So, let us assume that the large arcs referred to by Naylor are somewhere in between 160 and 120 degrees. As for the small arcs we can only speculate. To overcome these indeterminacies, I have simulated the motions of cork pendulums for a large range of parameters, by varying the mass of the cork bob and the size of the string (see section 4, for a more detailed discussion of the materials that Galileo would have used and of their characteristics). Further, I explored the motion of cork pendulums for large arcs of 60, 70, 80 degrees, and for smaller arcs of 10, 20, 30 degrees.

Figure 1.1, 1.2, 1.3 summarize the findings of a large number of computer-simulated motions. The figures show two curves, one for a cork pendulum starting from a *large* angle and one for an equal cork pendulum starting from a *small* arc at the same time. This format of presentation of the data allows for easier comparison. So, in figure 1, we have one pendulum starting from 60^o and one from 10^o, in figure 2, one pendulum starting from 70^o and one from 20^o, and in figure 3, one pendulum starting from 80^o and from 30^o. In all cases, after three complete oscillations, the pendulum going through larger arcs is late with respect to the other. The figure captions give further details about the characteristics of the cork mass and string.

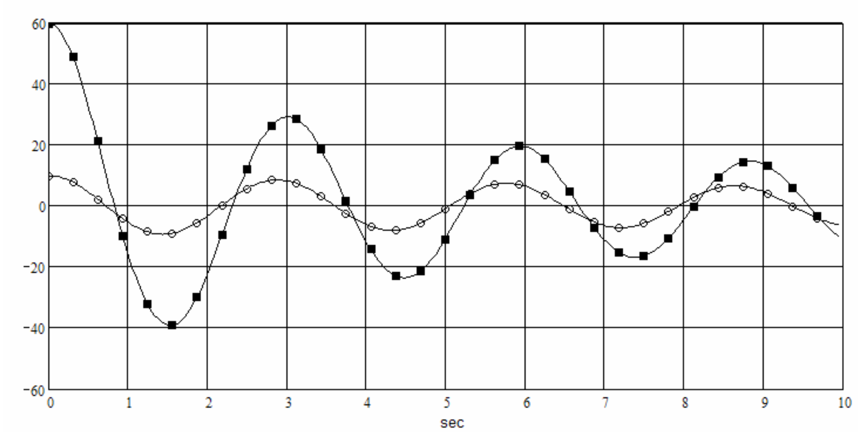

Fig. 1.1 One pendulum starting from 60^o (black square symbols) and one from 10^o (white circle symbols). The pendulum cork bob is 6 cm in diameter, weighing 29 gram. The string is 1 mm thick. By looking at the peaks of the angles one can see that after three oscillations the large arc bob (black square symbols) is slightly late.

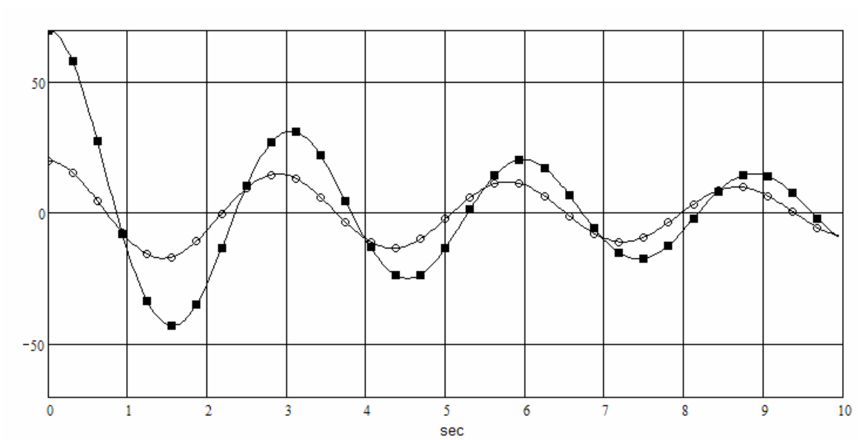

Fig. 1.2 One pendulum starting from 70^o (black square symbols) and one from 20^o (white circle symbols). The pendulum cork bob is 6 cm in diameter, weighing 29 gram. The string is 1 mm thick. By looking at the peaks of the angles one can see that after three oscillations the large arc bob (black square symbols) is once again slightly late, roughly ½ sec.

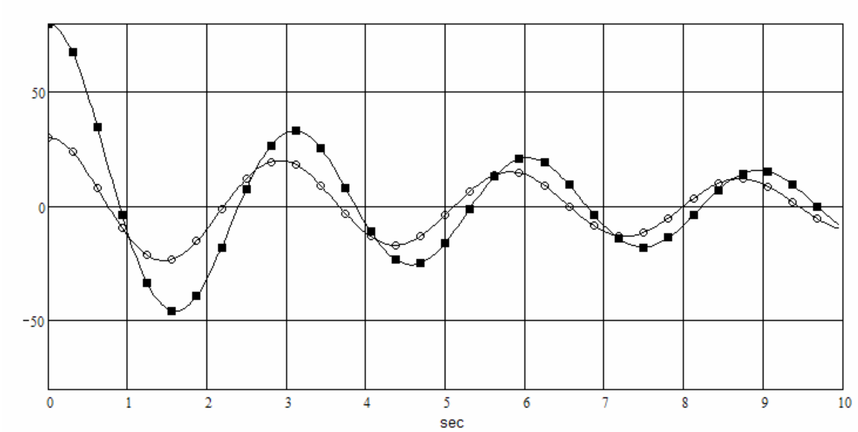

Fig. 1.3 One pendulum starting from 80° (black square symbols) and one from 30° (white circle symbols). The pendulum cork bob is 6 cm in diameter, weighing 29 gram. The string is 1 mm thick. By looking at the peaks of the angles one can see that after three oscillations the large arc bob (black square symbols) is once again slightly late, roughly ½ sec.

The simulated motions show consistency over a wide range of parameters. The cork pendulum swinging through large arcs is consistently slightly later than the other, about ½ sec. The order of magnitude of the delay is in accord with that of the time data reported by Naylor in table 1 for the cork pendulum, but by no means could I replicate in simulation the puzzling phenomenon seen by Naylor.

Finally, then, how could Naylor have obtained his puzzling results about cork pendulums? Here multiple hypotheses are possible. First, one must realize that air resistance is of the utmost importance in determining the slowing-down behaviour of the cork pendulum since cork is a very light material. Air resistance is roughly proportional to the square of the speed of the moving body.[1] Thus, it is hardly possible that a particular combination of cork bob and string will result in the pendulum being slowed down less severely when swinging from very large arcs than when swinging from small arcs. There must be some other reason for

Naylor's finding. I venture to hypothesize that the finding could be due to observational error. Indeed, Naylor himself admits that the measurements were more difficult in the case of the cork pendulums. The time data in table 1 show greater dispersion for the cork case. There is roughly a 1.3 sec difference between case 5 and case 6. This difference is much greater than the ½ sec delay between the two pendulums both shown in my simulations and estimated by Naylor. In other words, Naylor's observational error is much greater than the value he is trying to measure.

2. Weighing water with a balance

First of all, how did Naylor measure weights? Unfortunately, he does not report the way in which he weighed the water collected in the beaker. Judging from the fact that his experiments were carried out in the early 1970s, Naylor may have used a mechanical device, such as a scale, with a movable index on a graduated reference-scale (electronic devices were not cheaply available at that time). Given that Naylor reports his measurements in grams with two significant decimal digits after a decimal point, we must conclude that his apparatus could weigh with the precision of 1/100th of a gram.

Now, it seems that Naylor's claim that empirical evidence does not support Galileo's isochronism thesis rests on Naylor's personal assessment that a ~2 gram difference (average) between the brass bobs swinging along large and small arcs constitutes an empirical refutation of the isochronism hypothesis. In other words, Naylor's line of reasoning seems to be that since experiment does not show *equality* of measured times (that is, of weights of water collected in the beaker), then experiment does not support the isochronism hypothesis. But how could Galileo go about showing empirically *equality* of measured times?

For the sake of argument, let me first assume with Naylor that Galileo might have wanted to use a water-clock, as indeed Naylor did, in order to ascertain equality of times. I will subsequently suggest that this assumption makes no sense since the phenomenon of the *discrepancy*—which I will discuss below in reference to simulated examples, and in section 4 in reference to real experiments—would have clearly showed Galileo the pendulum's tendency towards *inequality* of times, that is, *inequality* of periodicities.

So, how could Galileo have gone about ascertaining empirically the equality of times measured with a water-clock? This question implies that we know what we mean by equality of measured times, that is, that we know what we mean by *equality of the weights* of water collected in the beaker. Since Naylor represents his weight measurements by numbers in the decimal system read off a graduated scale he obviously concludes that 32.58 gram is not equal to 30.49 gram. I agree.

However, Galileo did not use scales such as that presumably used by Naylor. Analytical balances with movable indexes, for instance, were certainly not in use in Galileo's time.[2] For all we know, Galileo would have used his remarkable *bilancetta*, a sort of small Roman balance, also known as steelyard, a precision instrument that he invented in his youth.[3] The *bilancetta* was excogitated by Galileo to measure specific gravities accurately. It was inspired by the so-called problem of Hiero's crown, whose solution—attributed to Archimedes in the Archimedean literary tradition—

Galileo found unsatisfactory. The *bilancetta* consisted of a thin beam, two braccia long, from one end of which hung the weight to be measured (in our case the beaker), while from the other hung the movable counterweight. It might have been hinged on a heavier support so as to leave the beam free to rotate around a pivot. Weighing with such an instrument requires balancing weight and movable counterweight by adjusting the position of the latter. But how do you actually do the measure reading?

The distance of the counterweight from the pivot gives the measure reading, at least in principle. In practice, to facilitate reading Galileo says that he coiled a thin metal wire around one of the beam's arms so that instead of taking the distance of the counterweight—with a compass, I imagine—from the pivot, an uncertain operation, he would count the number of coils by means of the clicking sound of a pointed stylus made to rub along the beam. Sound was more reliable than sight.

The crux of the whole process, though, is not so much the device by which the reading is performed as the very act of *balancing* the balance. It is at the moment when one is satisfied with the equilibrium of the balance that one takes the reading. But when does one reach that exquisite moment when one can say: "Equilibrium!"? What matters here is the visual perception of the state of indifferent equilibrium. But sight is definitely less reliable than hearing (as Galileo implicitly suggests by adopting the latter as means to count). The balance in equilibrium around the pivot is in a state of quasi-indifference to motion. It does not go up nor down decisively. It only displays *tendencies* to motion that can be seen the more vividly the less friction there is around the pivot. But there is a catch. The less friction there is around the pivot the more quickly the balance responds to disturbances, and thus the less comfortable the operator is in declaring “Equilibrium!”. On the other hand, the more friction there is around the pivot the greater the so-called “dead zone” of the balance, i.e., the operating zone within which the balance does not respond to adjustments in the position of the counterweight. In other words, the greater the dead zone the greater the uncertainty in the position of the counterweight, and hence in the measure being taken. A compromise is needed, not too much friction around the pivot, not too little. But there is more than this compromise. Indeed, the search for equality of weights cannot be a finite process. It is an infinite process.

So far I have said nothing about the relation of the weight to the counterweight. Obviously, in principle, one can have countless combinations of weight, counterweight and position of counterweight along the arm. For, the farther from the pivot the counterweight is placed, the greater its moment around the pivot (linearly, as we know from Archimedes’s law of the lever). This means that in principle we can have as many balances as we wish all doing the same job equally well. But not all of them will yield the same precision. Let us say that **q** is the weight of the counterweight which will balance the beaker’s weight **Q** when **q** is at distance **d** from the pivot. Let us also say that the thickness of a coil of wire is **δd**. Hence by adjusting the position of **q** by one coil, right or left of its tentative placement, the variation in total moment around the pivot (in absolute value) will be **q*δd**. But a greater **q** is needed to balance **Q** when we place **q** closer to the pivot, and of course a smaller **q** is needed to balance **Q** when we place **q** further from the pivot. In other words, the further we place **q** from the pivot the more precise the balance will be, in that it will allow us to produce a smaller and smaller variation in the total moment around the pivot, by simply adjusting the position of **q** by one coil, right or left of its tentative position.

The moral of this analysis should be clear. The choice of **q** is totally arbitrary. The smaller a given **q** is, the further from the pivot it needs to be placed to balance a given **Q**. The longer the balance's arm the more discriminating the balance is.

However, the process by which one can determine equality of weights can never end. It is an infinite process. Why? Once one has satisfied oneself that for given **Q1** (small arc pendulum), **Q2** (large arc pendulum), and **q** (counterweight), the balance remains in equilibrium, one needs to take a smaller **q** and repeat the operation, and so on ad infinitum, building longer and longer balances and picking smaller and smaller counterweights. The decision to stop at one point cannot but be arbitrary. The operator—and/or an accepted social praxis— decides that it is enough.

Thus, *equality* of two weights, **Q1**, **Q2**, corresponding to the two periods of two pendulums swinging along small and large arcs, is an abstraction. Like Godot it never comes. But it can be fixed in practice by acceptability norms of a pre-constituted social praxis that sanctions that an inequality may be taken as an acceptable equality.

On the other hand, it is quite possible that *inequality*, that is, disequilibrium (within the uncertainty limits of the dead zone due to friction, and/ or the discrimination limits fixed by a given **q*δd**), will show up at some point, thus putting an end to the infinite process. In Naylor's case, inequality shows up at the second decimal digit (the unit of the gram). However, Naylor does not explain why he progressed up to the second decimal digit. One might have been content with the first decimal digit (giving the tenth of gram) and proclaimed equality of the average weights. Was Naylor's decision an accident of the measuring apparatus he had at his disposal? Was it the accepted convention of the early 1970s that one had to declare a weight measurement with the discrimination of the gram? Was Naylor's choice due to the *Resolutions of the International System of Units*, fixed by the authority of the *Convention of the Metre*? Whatever his reasons the fact remains that Naylor is silent on them. Let me speculate, then.

As I take it, Naylor's implicit reasons seem to be that the water-clock plus *bilancetta*, when applied to the brass pendulums, might be a means of representing the tendency towards *inequality* of periodicities within: (a) the precision limits fixed by Galileo's two-braccia long *bilancetta*, or similar instrument, and (b) the acceptability norms of the social praxis contemporary with him. The norms might have been as follows: *If disequilibrium shows up in the bilancetta declare inequality. Pursue precision as far as the bilancetta allows. If disequilibrium does not show up declare equality.* Galileo only needed to translate the pendulum periodicities in terms of weights on the *bilancetta* and apply the above norms to see that pendulum isochronism was untenable. A 2-gram difference would sorely have popped up in the *bilancetta* with which Galileo claimed to discriminate weights down to 1/60th of a grain (1/24th of a scruple, or 1/576th of an ounce).

Two observations can be made that, I think, invalidate the above line of reasoning. First, the exceptional character of Galileo's *bilancetta*. Second, Galileo's water-clock.

The exceptional character of Galileo's bilancetta. Could the tacit norms for measuring equality of weights accepted in the social praxis contemporary with him encompass Galileo's *bilancetta*? I doubt very much since Galileo's *bilancetta* is an extraordinarily precise instrument, far exceeding anything that might have been in use at that time in terms of precision.

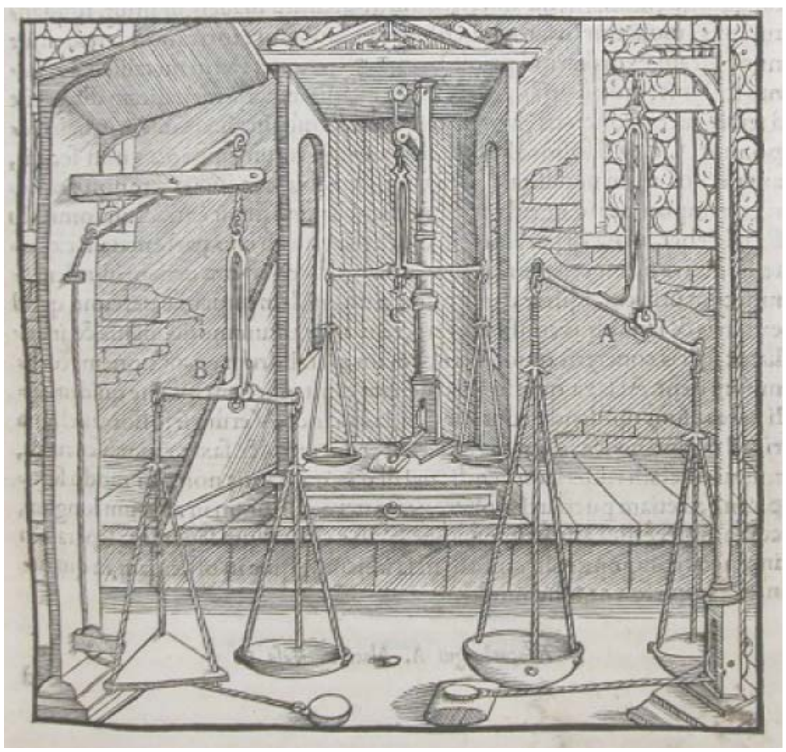

Fig. 2.1 Balances presumably in use during the sixteenth century for metal assaying as reported in the celebrated work, *De re metallica* (1556), by Georg Agricola.

Consider the balances in use during the sixteenth century for metal assaying as shown in Georg Agricola's *De re metallica* (1556).[4] First notice that they are equal-arm balances not steelyards with movable counterweight. This implies that the act of balancing with such a balance was carried out by means of weights to be placed on one of the pans in order to balance the weight to be measured. We can form an idea of the system of weights in use since Agricola has an illustration of a set of measure weights.[5] See figure 2.2.

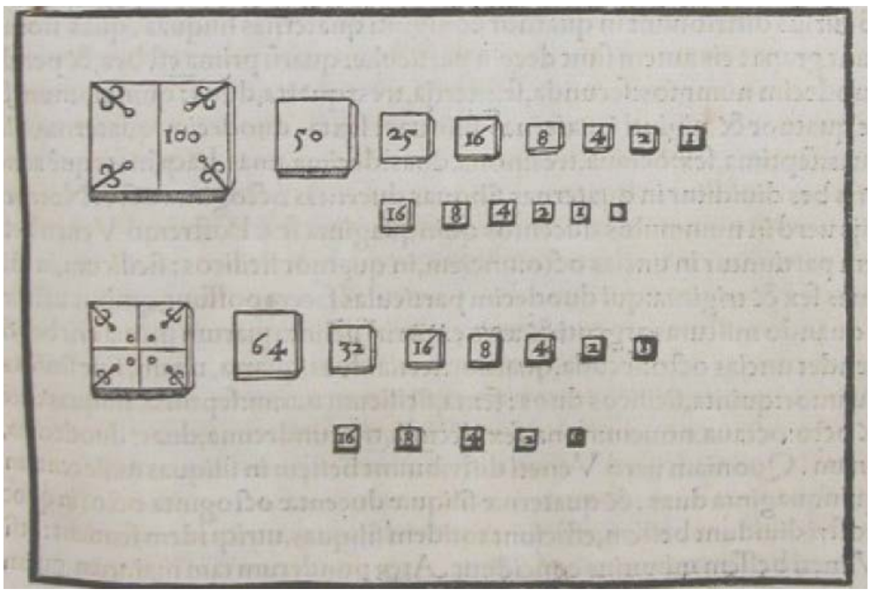

Fig. 2.2 Agricola's set of proof weights.

Agricola's discussion focuses on the intricate differences between a plethora of systems of measurement then in use. Agricola says nothing about issues of precision in measurement. However, an assaying balance is described in Lazarus Ercker's *Beschreibung allerfürnemisten Mineralischen Ertzt und Bergwercksarten* (1580) that seems more refined, and the making of a silver proof-grain is discussed.[6] Thus, we may assume that there was something of a pre-constituted praxis for metal assaying, in which an order of precision at which equality of weights could be declared, for instance in matters pertaining to comparison of the relative purity of silver and gold coins. All being considered, when Galileo decided to entitle his response to Orazio Grassi's *Libra astronomica ac philosophica* (1619) *The assayer* (1623), he might have implied that the assaying practice of precision was, by the then current standards, a sort of excellent reference standard, a much better precision than that obtainable by anybody using a *libra*. However, the *bilancetta* far exceeded that excellent order of precision. Indeed, Galileo's claim that he could push the *bilancetta* to the order of $1/60^{th}$ of a grain ($1/576^{th}$ of an ounce, almost two orders of magnitude smaller than the grain) must have seemed exorbitant to anybody accustomed to dealing with ounces, scruples and even grains. Thus, tacit norms for equality of weights (such as those accepted in the social praxis of metal assaying) could hardly have encompassed the *bilancetta*. That the *bilancetta* must have looked like an extraordinary instrument (by then current standards) is also suggested by three writings which have been preserved concerning its use and theory.[7]

To put it in nutshell, then, pursuit of precision as far as the *bilancetta* allows might not at all have been the *obvious* strategy to follow when it came to measuring equality of weights of the waterclock. Galileo had to constitute his own arbitrary norms for declaring equality, and blindly stop the infinite pursuit of precision in a praxis-vacuum for the *bilancetta*. L. Niinistö reports that by the "early 18^{th} century the accuracy [*of analytical balances*] improved to a few milligrams but, for instance, Joseph Black (1728-1799), who contributed to the development of balances and quantitative analytical chemistry, reported analytical data to the nearest grain only".[8] So, even when better precision is available, there may not be an immediate need to push the declaration of equality/non-equality of weights to the best available level of precision. The convention of a *false* equality may still be perfectly acceptable praxis.

Galileo's water-clock. The puzzle concerning the water-clock, however, is that Galileo does not need to measure pendulum periodicities with a water-clock. As I anticipated, their tendency towards *inequality* is all too readily apparent in the discrepancy phenomenon. Indeed, the question is exactly why Galileo never mentions the use of a water-clock in connection with the pendulum, instead showing indifference to this possibility. (The only exception, quoted but misinterpreted by Naylor, is a late letter in which Galileo suggests to his correspondent a procedure to verify that a heavy body descending from a height of 100 braccia will fall in approximately 5 seconds).[9] The answer is twofold. Firstly, the pendulum requires the shaping of a novel act of "balancing" periodicities, and secondly no established norms of acceptance for their equality (unlike the equality of weights on a balance) are available in contemporary praxis. That the latter must be the case it seems obvious to me since Galileo invented the pendulum as a time-dividing instrument of scientific practice. In the paper, the reader will see my discussion of the former— the novel act of "balancing"— and why it barred translating pendulum periodicities in terms of weights on the *bilancetta*.

3. The pendulum apparatus

In the collective book reporting their experiments (published in 1666), the experimenters of the *Accademia del Cimento* presented a pendulum in the form of gallows-like structure. It was depicted in the historiated capital of the first paragraph of the section on time measurements. It struck me. It is a simple but elegant structure. In the marginal note, we read, "Experiences that require the exact measure of time [*Esperienze, che richiedono la misura esatta del tempo*]".

The real instrument, built and used by the *Accademici*, is then depicted in detail in a full-page table (Fig. 3.1). I was inspired by the pendulum of the *Accademici*, which might in turn have been inspired by Galileo via the mediation of Galileo's pupil, Vincenzo Viviani, himself a member of the Cimento Academy.

Unfortunately the only suggestion we get from Galileo concerning his pendulum set-up is the diagram accompanying the 1602 letter to Guido Ubaldo (cf. T1). The arrangement suggested in T1 is unconvincing. It is of course quite possible that Galileo initially had two 2-3 braccia pendulums adjacent to each other on a wall. But if we consider that later on he refers to 4-5 braccia pendulums, I think that the T1 arrangement becomes implausible. In order for the pendulums not to hinder each other, they would have to be hung at a distance such as to allow their full extension. This means a wall of at least 20 braccia, about 7 meters, which would then require the observer to stand at a considerable distance, thus making the observations rather difficult. Further, the 1602 letter is preserved only in a third-hand copy of the nineteenth century. The diagram of the two pendulums looks suspicious since, in the text, Galileo says that one of the pendulums is removed from the perpendicular "a lot [*assai*]", while the other is removed "very little [*pochissimo*]", yet the two arcs drawn in the diagram are almost of the same amplitude. Moreover, it is possible that, even if accurate, the diagram only had an illustrative function, and was not intended to describe a real set-up.

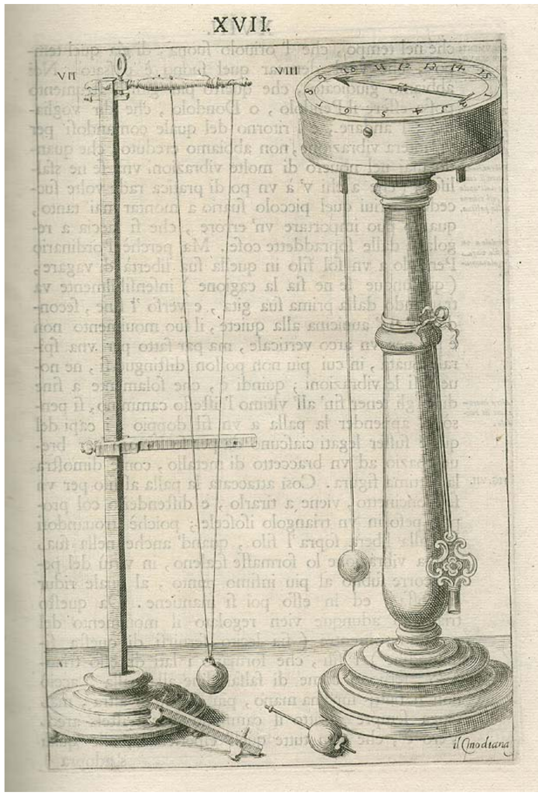

Fig. 3.1 The pendulum used by the *Accademici del Cimento*. The shape of the pendulum support (left) initially inspired my gallows-like structure. The *Accademici*'s pendulum, though, seems to be a very small instrument. See L. Magalotti, *Saggi di naturali esperienze fatte nell'Accademia del cimento sotto la protezione del serenissimo principe Leopoldo di Toscana e descritte dal segretario di essa accademia*, Florence, 1666, p. 21.

I therefore came to the conclusion that a different structure might have been used by Galileo. The *Accademici* offered me a plausible solution. The gallows-like structure has the advantage of allowing for easier observation of the oscillations of two pendulums relative to each other. Since isochronism and synchronism, as explained in section 2, were the focus of the investigation, this seemed an ideal arrangement for the pendulums.

I modified a wooden easel in order to build a structure which could easily accommodate the electronics needed to acquire the data for the computer model of the pendulum. The suspicion that a phenomenon of mechanical coupling might be lurking in structures like this, as I had already predicted on the basis of simple computer models, suggested that I build a light horizontal arm which could subsequently be stiffened by means of cables fixed to the walls of the lab. This solution worked well and allowed for a certain freedom in adjusting the cables, so as to control the amount of residual flexibility left in the structure.

The data acquisition system is based on miniature, high precision load cells capable of measuring the tension in the strings of the pendulums. Ideally one could measure directly the angular position of the pendulums, but in practice, since the strings are not rigid, easy and economically affordable solutions, based on angular position sensors, such as, for instance, rotation potentiometers, are unavailable. On the other hand, the measurement of tension made possible by load cells is very precise.

Fig. 3.2 The horizontal arm, made of Plexiglas, showing the two load cells, above it, two hemp strings directly connected to the load cells, a few hooks, and the stiffening cables fixing the horizontal arm to the wall of the lab.

The load cells are then connected to a data acquisition system, not shown above, which converts the analogical signals from the cells into a numerical format. The acquisition system is connected to a laptop computer, on which special software allows for further elaboration of the acquired data.

The load cells I used are capable of reading loads up to 100 grams, with virtually infinite sensitivity, only limited by the finite length, i.e., number of bits, of the output from the analogue to digital converter inside the data acquisition system (a 14-bit A/D converter in the system I used). The range and high precision of the load cells is perfectly suited to measurements with pendulums carrying light lead balls, in the range of the 1-2 ounces that Galileo might have used.

The tension generated in the string of the pendulum can easily be calculated while simulating the mathematical model, so that an accurate comparison can then be made between acquired tension data and calculated tension data. This comparison is the basis for the fine-tuning of the aerodynamic parameters of the mathematical model of the pendulum (see the discussion of the pendulum computer model in the RGE, Appendix 1).

As for the materials of strings and balls, let us first note that often Galileo refers to the strings of his pendulums as “spago” or “spaghetto”. A “spago” would presumably have been made of either hemp or linen. The strings of my pendulums were thin and made of natural hemp or of linen (about 1.5 mm in diameter, though diameter varies, especially for hemp strings). Natural hemp tends to be more fluffy and knotty than linen, so I initially thought that this might be a factor in determining aerodynamic forces, but it turned out that there is no appreciable difference between strings having more or less fluff or little irregularities such as small knots. In fact the aerodynamic forces are due basically to the length and thickness of the string. The 1623 edition of the *Vocabolario degli Accademici della Crusca*, says that “canapa [hemp]” was used to make “corde, funi, e anche tele”, and that a “spago” is a “funicella”, i.e., a thin “fune”, which therefore suggests to me that hemp would likely have been used by Galileo for his “spago” or “spaghetto”.[10]

As far as lead balls are concerned, I investigated the possibility that Galileo used musket bullets, since they were roughly the size of 1-2 ounces (as he indicates on one occasion, in the experiment of the pendulum twisting around the peg is 1-2 ounces, as we have seen). Bullets would have been easily available to him, I guess, since he was in contact with military students and engineers, especially in Padua. I purchased a few historic musket bullets, relics of the English Civil War, and noticed that they tend to be rather irregular both in size and weight. If this is any indication of the quality of early seventeenth-century bullets, and if Galileo used anything like bullets, then we must make allowances for possible differences in what he tends to call lead “balls [*palle*]”. On the other hand, Galileo might also have cast his own lead balls. The technology would have been available to him since the melting temperature of lead is not too high.

At any rate, I eventually decided to do my experiments with modern lead balls, which are commonly used in fishing. They come in different sizes and have a hook, so that it is easy to experiment with different weights. I noticed no difference between a modern lead ball and a historic bullet of roughly the same size and weight, though the bullet, not having a hook, has to be tied to the string differently. As for the cork balls, I need to point out that, since cork is a natural material somewhat variable in specific weight, it is impossible to say exactly how large Galileo's cork balls would have been, even when the weight can be estimated with some accuracy (and the same could be repeated of early modern lead, though perhaps to a lesser extent).

Finally, as for the video material, I used a camera with wide-angle lens to shoot the videos in the lab, since the amplitude of the oscillations of the pendulums can be very large. Subsequently, the videos were downloaded into a laptop computer, reviewed, edited, and finally converted to a suitable format for delivery via the internet or other digital media.

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[1] Cf. RGE, pp. 215-219, for a detailed discussion of the computer model of the pendulum.

[2] Cf., for example, L. Niinistö, 'Analytical instrumentation in the 18th century', *Fresenius' Journal of Analytical Chemistry* (1990), 337, 213-217.

[3] EN, I, pp. 209-228.

[4] Georg Agricola, *De re metallica*, Basel, 1556, p. 207.

[5] Agricola 1556, op.cit. (16), p. 205.

[6] See L. Ercker's description, in English translation, in J. Pettus, *Laws of art and nature, in knowing, judging, assaying, fining, refining and inlarging the bodies of confined metals*, London, 1686, pp. 86-98. Cf. L. Ercker, *Beschreibung allerfürnemisten Mineralischen Ertzt und Bergwercksarte*n, Frankfurt am Main, 1580, p. 79, for the construction drawing of the balance. This balance was reconstructed in 1962 to the instructions given in Ercker's "Treatise", and a picture is avaialble at: http://www.scienceandsociety.co.uk .

[7] See Galileo Galilei, *Opere di Galileo Galilei Nobile Fiorentino*, IV, Milan, 1810, pp. 243-270. These writings (not by Galileo) were not published in EN.

[8] Niinistö 1990, op.cit. (2), p. 214.

[9] EN, XVIII, pp. 75-79. The procedure suggested by Galileo is as follows. First, he appeals to a theorem he published in *Two new sciences*, which proves that once we know the time of fall along an inclined plane of any length we can easily compute the time along any vertical fall. Then, we need to ascertain the length of one second in reference to the 24 hour period of rotation of the earth. To achieve that determination of the "sidereal second" Galileo argues that all that is needed is that a few friends let a pendulum swing and count the number of oscillations for the entire duration of the time required for a fixed star to return to cross a fiduciary mark such as a chimney, or something like that, after a day. Then, by the rule expounded in *Two new sciences* that relates pendulums' periods to their strings' lengths one can easily compute the length of the pendulum which will beat the second, or any smaller, sexagesimal fraction than the second that we might wish. At this point of the letter, Galileo goes on to say: "It is true that we can pass to more exact measurements after seeing and observing what is the flow of water in a thin pipe, since by collecting the water and weighing that which will flow, say, in one minute, if we then weigh the [*water*] flowed during the time [*taken by the mobile*] to fall along the channel [*of the inclined plane*], we will be able to find the most exact measure and quantity of that time if we make use of so precise a balance that it can discriminate 1/60th of a grain". Here Galileo alludes to the waterclock— I agree with Naylor on this count. But the question is why the waterclock is needed to perfect this measurement, and here I must disagree with Naylor. Not because the pendulum cannot in principle achieve any precision you can possibly desire, as already mentioned by Galileo. But because the fact is that, although a shorter and shorter pendulum calculated as indicated earlier in the letter will in principle beat any sexagesimal fraction of the second, practically 1/60th of a second (the next to the second subdivision in the sexagesimal order) is way too fast to observe (60Hz is in fact an audible frequency for the normal human ear). Now, the descent of the body along an inclined plane (of 12 braccia, for example), the timing of which is required for us to arrive at the time of fall along 100 vertical braccia, can take any value of the order of a few seconds. Hence to achieve adequate precision, Galileo suggests, an accurate estimate is required of the time in terms of at least some fraction of the second. The waterclock, then, is used here first to measure how much water will flow in one minute (timed by sixty oscillations of the

pendulum which will beat the second), and subsequently to time the descent along the inclined plane (as already expounded by Galileo in *Two new sciences*). The waterclock is useful for practical purposes, in order to improve the precision limit of the pendulum beating the second. Its use is suggested by Galileo to overcome the deficiency of the human eye in observing oscillations faster than one second, not to time the period of the pendulum, as was erroneously thought by Naylor. Cf. R. Naylor, 'Galileo's simple pendulum', *Physis* (1974), **16**, 23-46, on p. 29.

[10] Cf. Accademici della Crusca, *Vocabolario degli Accademici della Crusca*, 2nd ed., Venice, 1623, ad vocem.